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Parameterization of the harmonic content of the complex MPI signal of magnetic tracers using a set of polynomial coefficients

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ABSTRACT

In our work, we investigate the possibility of a polynomial interpolation of the Magnetic Particle Imaging (MPI) signal to parameterize the harmonic content of the signal to analyze the imaging quality at a limited number of harmonics. The traditional Taylor expansion of the magnetization curve is limited by a radius of convergence which is exceeded by the physical properties of real nanoparticle ensembles and MPI setups. This led us to the derivation of an alternative method using polynomial interpolation. Our method enabled us to express MPI signals based on the analytical Langevin function as well as numerical simulated data using other magnetization functions and a phase shift in relation to the excitation signal. The argument range of the interpolation function is wide enough to cover real cases. We conclude that the polynomial interpolation method represents a versatile tool to describe the relation between the magnetization curve and the MPI signals.

1. Introduction

Magnetic Particle Imaging (MPI) is an imaging method to visualize magnetic nanoparticles (MNP), so-called MPI tracers, in the body. It is based on the detection of higher harmonics induced by the nonlinear magnetization curve and the time lag of the magnetic moment of an ensemble of MPI tracers in an oscillating magnetic field. An additional gradient field ensures specific harmonics of the tracer signal at every tracer location. Decomposing the total MPI signal into the characteristic contribution from each voxel allows a spatial reconstruction of the MNP amount.

To analyze the MPI signal one approach is to use a polynomial expression of the magnetization function modeled by the Langevin function based on the Taylor expansion [1,2]. This approach has the main advantage that the creation of harmonics can easily be observed via analysis of the polynomial coefficients and basically would be a powerful tool to investigate the connection between magnetization curve and MPI spectrum. Yet, there are disadvantages: the approach is not capable of projecting the time lag of the response of MNP and therefore could only describe (quasi)static signals. Furthermore, it is restricted to the use of the Langevin equation where it exhibits a radius of convergence of $(-\pi,+\pi)$ for the expansion, caused by the polynomial description of $f(x)=\coth(x)$ [2].

We present a new approach to interpolate the measured or simulated MPI signal using a set of polynomial coefficients. Our approach enables us to express the phase shift of the MPI signals and is not limited to a small radius of convergence.

In this paper, we will describe how to derive the set of polynomial coefficients for a given number of polynomials and how to apply this method to measured and simulated data.

2. Method

As a very simple approximation the $M(H)$ behavior of MPI tracers can be described by the Langevin function $L(x)$:

$$L(x)= \coth(x) - 1/x \tag{1}$$

with

$$x:=\xi=\frac{mH\mu_0}{k_BT} \tag{2}$$

Here, m is the magnetic moment, H is the magnetic field strength, μ_0 is the magnetic vacuum permeability, k_B is the Boltzmann constant and T is the temperature. The magnitude of x is directly proportional to the magnetic moment m and the magnetic field strength H and therefore determines the steepness of the $M(H)$ curve of an MNP ensemble at $H=0$. It can be seen that this model does not consider the magnetic anisotropy and therefore the time lag of the magnetic response is not taken into account. Still for some applications this model is sufficient.

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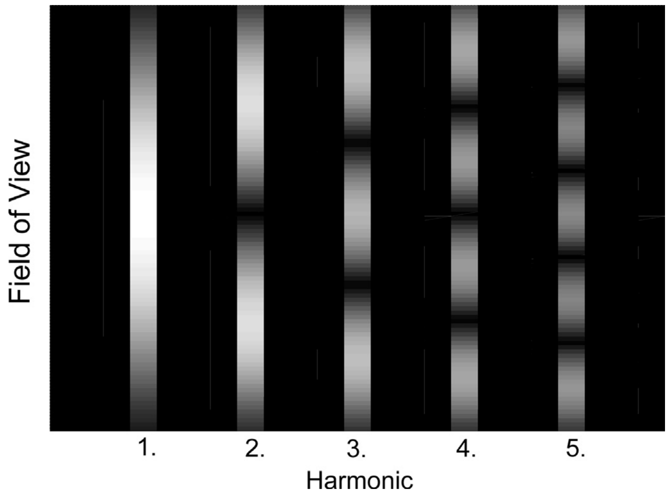

Fig. 1. Signal intensity in dependence on the position and harmonic number.

One of the most basic methods to describe functions is the polynomial description by a Taylor expansion. Applying this expansion on the Langevin function $L(x)$ around the point $x=0$ yields

$$L_T(x) = \frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} - \frac{x^7}{4725} + \frac{2x^9}{93555} + O(x^{11}) \tag{3}$$

Since MPI uses an oscillating drive field, x can be written as $x=x_{\max}\sin(\omega t)$. Using the relation

$$\sin(x)^n = \sum_{n=0}^{N} K_n \sin(nx) \tag{4}$$

it is possible to control the number of harmonics in the process which are later used in the image reconstruction.

This limitation of harmonic content is needed for simulations regarding the relation between image quality (i.e. the resolution) and the amount of available harmonics for image reconstruction, as we expect the image quality to be dependent on the amount of harmonics above noise [3].

Fig. 1 depicts the signal characteristics of the first five harmonics at different locations of the field of view for the Langevin function. Every harmonic has a limited number of maxima and minima. Our hypothesis is that the amount of maxima and minima of the highest measurable harmonic above noise is related to the achievable resolution and the display of more MNP maxima than harmonic maxima is not possible.

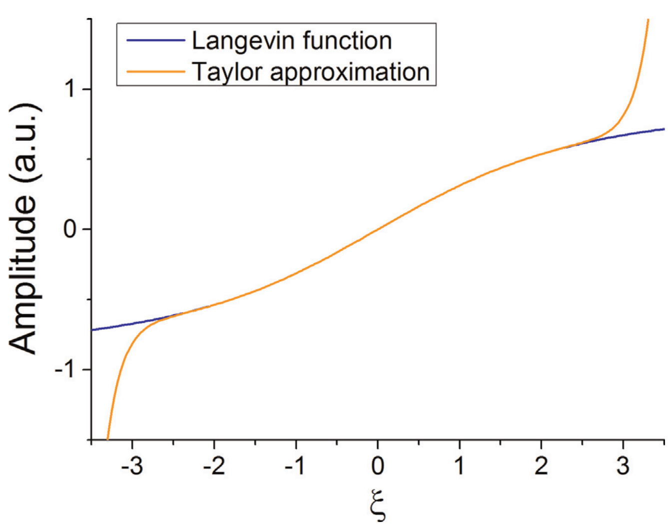

Fig. 2. Amplitude of the Langevin function compared to the amplitude of the corresponding Taylor approximation in the range $\xi=[-3.5,+3.5]$.

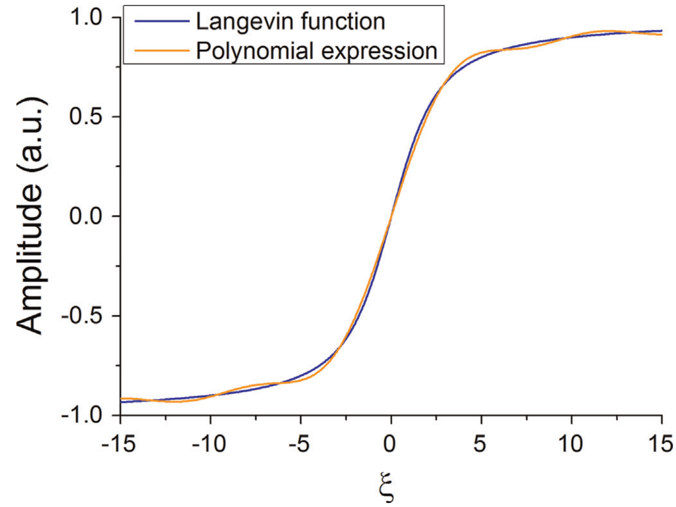

Fig. 3. Comparison of the Langevin function and its corresponding polynomial expression with $N=7$ and $\xi=[-15,+15]$.

A real $M(H)$ curve differs from the simplistic Langevin model. Furthermore, the MPI signal exhibits a time lag in magnetization with respect to the drive field and, as mentioned before, the Taylor expansion method is only applicable in a radius of convergence of $\pm\pi$ (Fig. 2).

To preserve the advantages of a polynomial description of the MPI signal we describe the magnetization $M(x(H))=M(x(H_m))=M(x(H_0\ldots H_G))$ as a system of linear equations:

$$\begin{pmatrix} M(x_0) \\ \vdots \\ \vdots \\ M(x_G) \end{pmatrix}^T = \begin{pmatrix} a_0 \\ \vdots \\ \vdots \\ a_N \end{pmatrix}^T \begin{bmatrix} x_0{}^{2\cdot 0+1} & \cdots & \cdots & x_G{}^{2\cdot 0+1} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ x_0{}^{2\cdot N+1} & \cdots & \cdots & x_M{}^{2\cdot N+1} \end{bmatrix} \tag{5}$$

with $M(x)$ as the measured or calculated magnetization, the prefactors a and the polynomials x^{2n+1}. Solving this linear equations system, one can obtain the vector of coefficients a_n that describes the magnetization.

This method is a suitable tool to describe the static magnetization or magnetic moment either from simulations or from actual measurements. The approach reaches its limit at the description of a dynamic magnetic moment that is phase shifted in relation to the excitation signal. In order to include phase shifts we extend Eq. (5) by a set of cosine terms:

$$\begin{pmatrix} m(x_1) \\ \vdots \\ m(x_G) \end{pmatrix}^T = \begin{pmatrix} a_1 \\ \vdots \\ b_N \end{pmatrix}^T \begin{bmatrix} \sin(x_1)^1 & \cdots & \cdots & \sin(x_G)^1 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \sin(x_1)^N & \cdots & \cdots & \sin(x_G)^N \\ \cos(x_1)^1 & \cdots & \cdots & \cos(x_G)^1 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \cos(x_1)^N & \cdots & \cdots & \cos(x_G)^N \end{bmatrix} \tag{6}$$

This yields the possibility to not only parameterize static but also the dynamic magnetic moment as it is needed in a proper MPI simulation.

Then, the description of the signal is given by

$$m(x) = \sum_{n=1}^{N} a_n \sin(x)^n + b_n \cos(x)^n \tag{7}$$

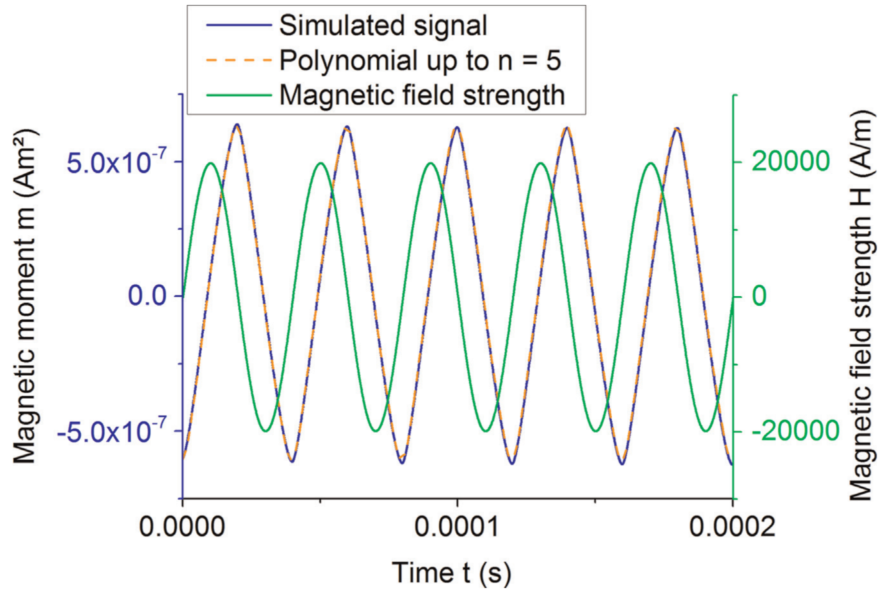

Fig. 4. Comparison of a time shifted simulated magnetic moment and its corresponding polynomial expression in relation to the excitation signal.

Using Eq. (4) and its cosine counterpart Eq. (7) can be rewritten as

$$m_{\tilde{N}}(x) = \sum_{n=1}^{N} K_{n,1} \sin(nx) + K_{n,2} \cos(nx) \tag{8}$$

Summing up the fractions of $K_{n,1}$ and $K_{n,2}$ for a given n of all polynomials the overall components for the n-th harmonic $C_{n,1}$ and $C_{n,2}$ are calculated, which can be transformed to the classic harmonic notation.

$$\begin{aligned} \sum_{i=1}^{n} K_{i,1} \sin(nx) + K_{i,2} \cos(nx) \\ = C_{n,1} \sin(nx) + C_{n,2} \cos(nx) \\ = C_n \sin(nx + \varphi) \end{aligned} \tag{9}$$

We therefore conclude that a contribution of a certain number of polynomials corresponds to the same number of harmonics in the MPI signal.

3. Simulation results

Using this method, Fig. 3 depicts the Langevin function compared to its resulting parameterization with seven polynomials and $\xi = [-15,15]$ with $M(\xi) = 2.7 \sin(\pi\xi/2\xi_{\max}) - 5.6 \sin^3(\pi\xi/2\xi_{\max}) + 6.7 \sin^5(\pi\xi/2\xi_{\max}) - 2.9 \sin^7(\pi\xi/2\xi_{\max})$. Note that the polynomials with even exponent are set to zero due to the symmetric nature of the $M(H)$ curve, comparable to the harmonics. It can be seen that the constraints concerning the radius of convergence can be bypassed and a parameterization of the signal up to $\xi = \pm 15$ is not a problem anymore. In comparison of Figs. 2 and 3 one can see that the classic Taylor approximation results in a very close fit inside the radius of convergence and diverged outside of the radius. On the other side we can see a slight oscillation of the parameterization using our method over the whole range described here, but without complete divergence of the interpolation.

This indicates a need for more polynomials for a better description of the Langevin function up to $\xi = \pm 15$ which means that this signal is not suitably described with only seven harmonics.

Fig. 4 depicts a simulated magnetic moment which is time shifted in relation to the magnetic excitation field. Using our extended model, we can describe the simulated magnetic moment using sine and cosine polynomials up to $\sin^5$ and $\cos^5$, respectively.

It can be seen that the parameterization describes the simulated signal nearly perfectly with polynomials up to the fifth degree. Due to the direct correlation between polynomials and harmonics the deviation in the parameterization gives us an estimation of the potential imaging quality of the tracer since a high deviation indicates the need for higher polynomials (and therefore harmonics) to describe the signal.

4. Discussion

We proposed a method to describe the measured or simulated signal of the magnetic moment using a set of polynomials for interpolation of the $M(H)$ curve. With our method we were able to bypass the limitations of the Taylor expansion which is usually used to describe a function as a series.

Thus, we interpolated the Langevin function without the aforementioned limitation of the radius of convergence as well as numerically simulated magnetization curves.

This method gives us the opportunity to manually limit the amount of harmonics via the amount of polynomials. This manual limitation enables us to draw first conclusions about the imaging quality of the tracer.

We believe that this method might be suitable for the parameterization of simulation data and will be a tool for our future work on the connection of measureable harmonics and image resolution.

Acknowledgments

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References

[1] T. Knopp, T. Buzug, Magnetic Particle Imaging – An Introduction to Imaging Principles and Scanner Instrumentation, Springer, Heidelberg, 32–35.
[2] J.-P. Gehrcke, Characterization of the Magnetic Particle Imaging Signal Based on Theory, Simulation, and Experiment, Master thesis, , 2010.
[3] J. Rahmer, Signal encoding in magnetic particle imaging: properties of the system function, BMC Med. Imaging 9 (2009).