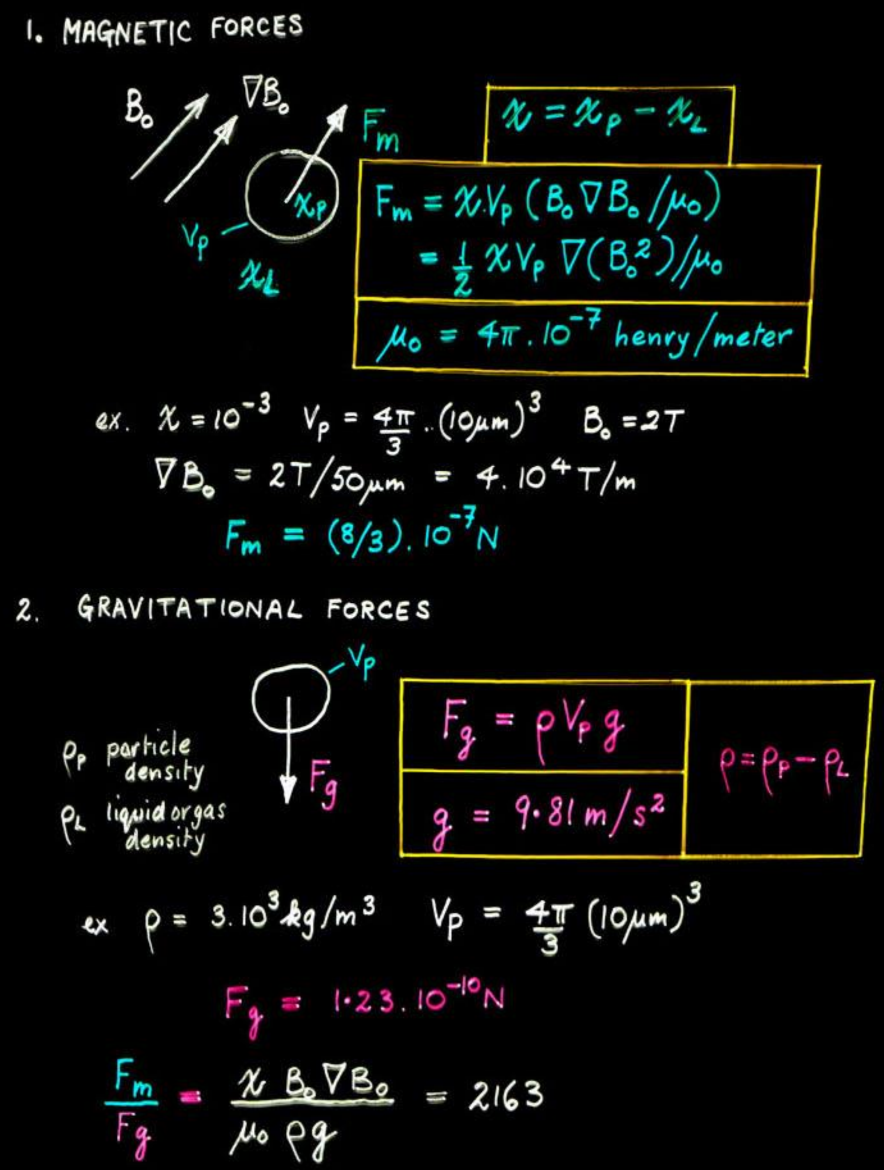
Capture of Small Particles on Magnetic Single Wires

J.H.P. Watson

University of Southampton, Southampton, UK
jhpw@soton.ac.uk

Forces on small particles



3 fluid forces

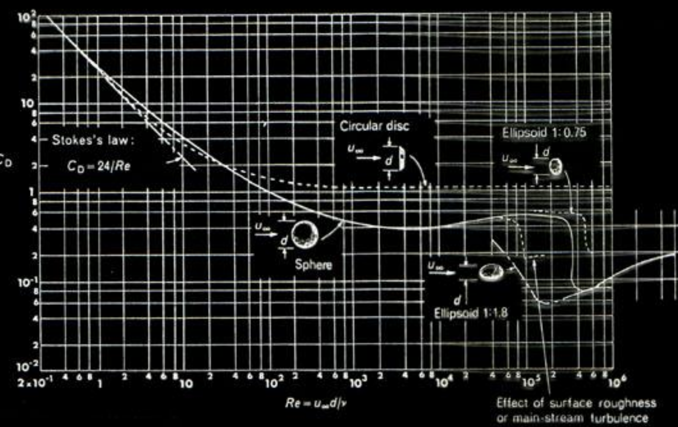
V – relative velocity of the body and the fluid of density ρ_L and viscosity η

$F_d = C_D \left(\frac{\rho_L V^2}{2}\right) A_p$	$R_e = \frac{V d \rho_L}{\eta}$

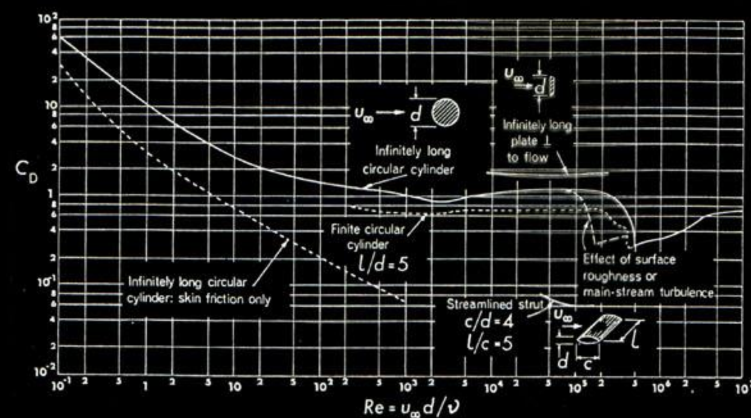
In the Stokes region $C_D = 24/Re$ so for a sphere with radius b then

$F_d = 6\pi \eta b V$	Stokes's formula

Ex $\eta = 10^{-3}$ Pa-s $\rho_L = 10^3$ kg/m^3 $V = 1$m/s $b = 10\mu$m
$F_d \simeq 3.5 \cdot 10^{-7}$N $Re = 20$

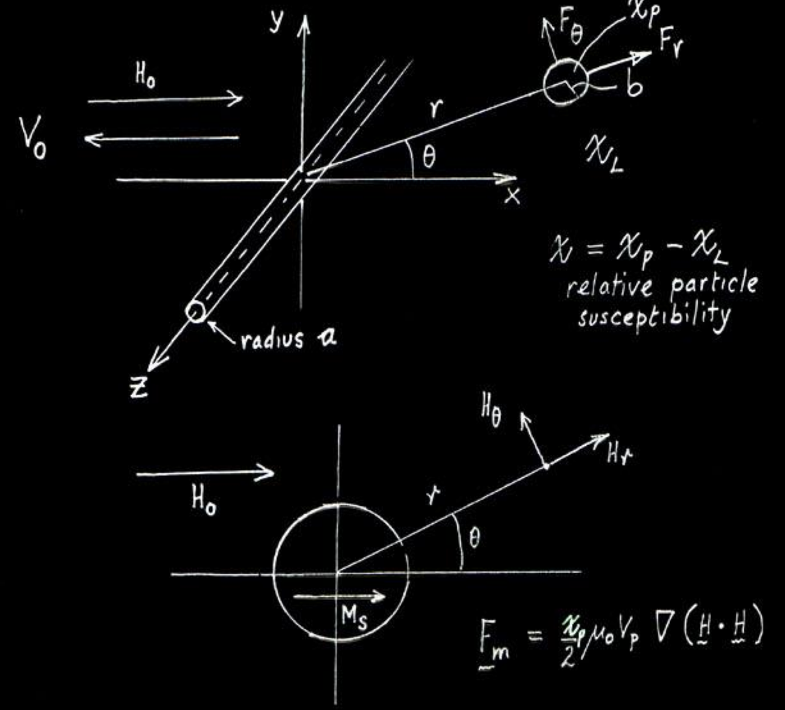


3-dimensional bodies



2-dimensional bodies

Force between wire and a particle



$$F_r = -6\pi\eta b V_m \left(K/r_a^{5} + \cos 2\theta / r_a^{3} \right)$$

$$F_\theta = -6\pi\eta b V_m \sin 2\theta / r_a^{3}$$

$$r_a = r/a$$

Force between wire and a particle

Neglecting particle inertia and gravitation

$$F_d + F_m = 0$$

For flow around a wire of circular cross-section in the viscousless approximation.

$$V_r = -V_0\left(1 - 1/r_a^2\right)\cos\theta$$

$$V_\theta = V_0\left(1 + 1/r_a^2\right)\sin\theta$$

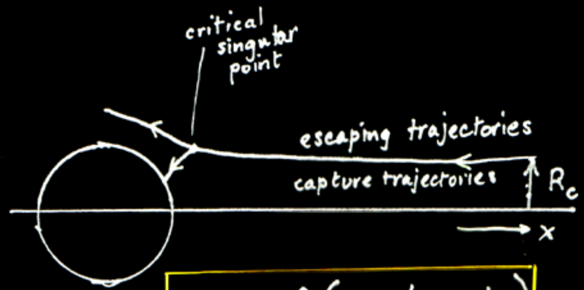
$$(F_d)_r = 6\pi\eta b\left(a\frac{dr_a}{dt} - V_r\right)$$

$$(F_d)_\theta = 6\pi\eta b\left(a r_a\frac{d\theta}{dt} - V_\theta\right)$$

$$a\frac{dr_a}{dt} = V_0\left(1 - 1/r_a^2\right)\cos\theta - V_m\left(K/r_a^5 + \cos 2\theta/r_a^3\right)$$

$$a r_a\frac{d\theta}{dt} = -V_0\left(1 + 1/r_a^2\right)\sin\theta - V_m \sin 2\theta/r_a^3$$

Force between wire and a particle



$$R_c = f\left(V_m/V_0,\ K\right)$$

R_c is capture radius or critical entering co-ordinate.

$V_m = \frac{2}{9}\left(\mu_0 \chi b^2 \frac{M_s}{a} \cdot \frac{H_0}{\eta}\right)$	Magnetic Velocity
$K = M_s/2H_0$	Short range force parameter

MKS $M_s \equiv T$. $V_m = \frac{2}{9} \chi b^2 \frac{M_s}{a} \frac{B_0}{\eta} \frac{}{\mu_0})$ SI M_s amp/metre

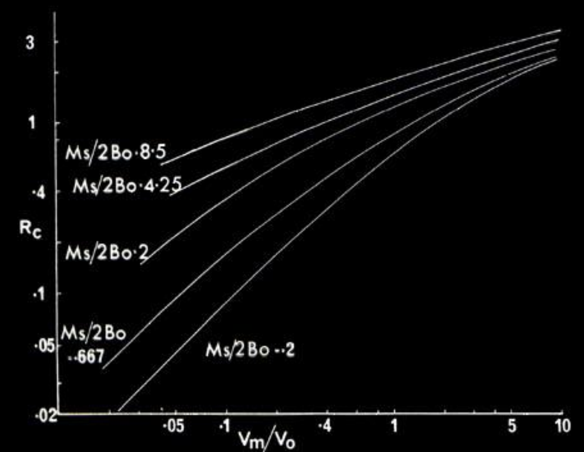
$$V_m = \frac{2}{9}\left(\mu_0 M_p \frac{M_s}{a\eta}\right)$$

Magnetic Velocity for superparamagnetic particles where M_p is the saturation magnetisation of the particle

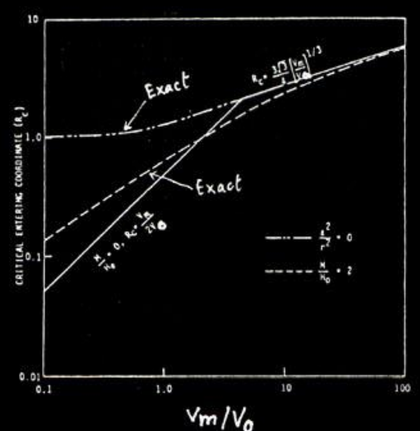
The capture radius R_c

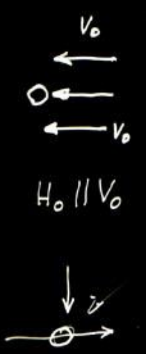
Neglecting the short range term for a clean wire

$$R_c = \begin{cases} V_m/2V_0 & \text{for } V_m/V_0 \leq \sqrt{2} \\ \frac{3\sqrt{3}}{4}\left(\frac{V_m}{V_0}\right)^{1/3} & \text{for } V_m/V_0 > 1{\cdot}0 \end{cases}$$



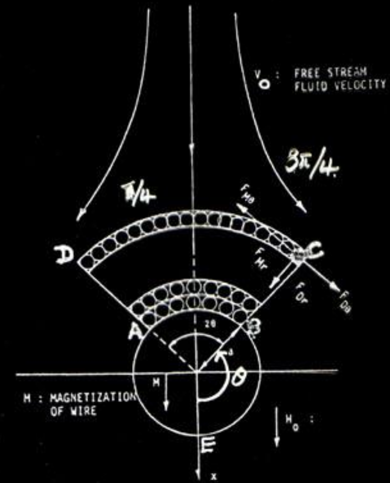
$M_s/2B_0 > 1$ only possible for materials showing magnetic hysteresis





Case where $H_o \perp V_o$ gives similar values of R_c

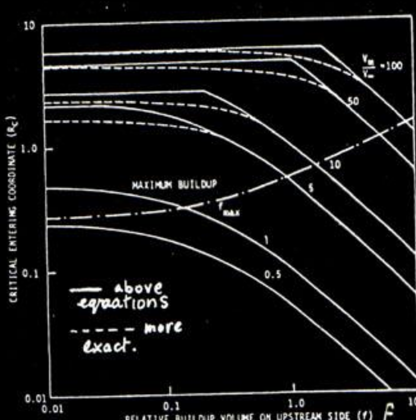
The capture radius R_c



$$R_c = \begin{cases} \dfrac{V_m}{2V_0}\left(\dfrac{1}{4f+1}\right) & \text{if } f \geq f_a \\ \dfrac{3\sqrt{3}}{4}\left(\dfrac{V_m}{V_0}\right)^{1/3} & \text{if } f < f_a \end{cases}$$

$$f_a = \frac{1}{4}\left\{\frac{2}{3\sqrt{3}}\left(\frac{V_m}{V_0}\right)^{2/3} - 1\right\}$$

where $f = \dfrac{\text{area ABCD}}{\pi a^2}$ β



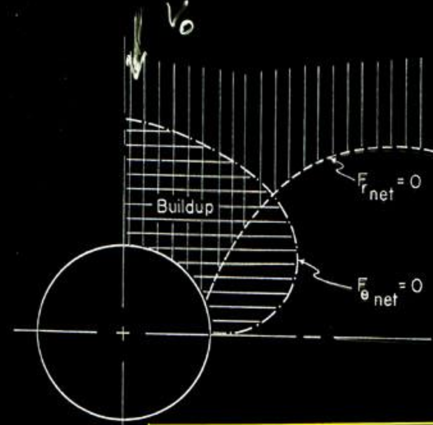
R_c calculated here gives the trajectories which will collide with the wire with the deposit f. The question of whether the particle will stick has not been considered.

$$\frac{\delta}{a} = \sqrt{\frac{\pi}{R_e}}$$

Assuming potential flow around the central particle build-up the limit of build-up is determined when $F_d)_\theta > F_m)_\theta$

$$f_{max} = \frac{1}{\pi}\left(\frac{V_m}{V_0}\right)^{1/3}\left(\frac{3\delta}{b}\right)^{2/3}\left[\int_{3\pi/4}^{\pi} |-\cos\theta|^{2/3}\, d\theta - \frac{1}{4}\right]$$

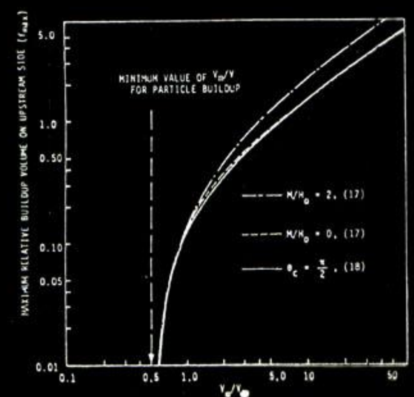
The capture radius R_c



Nesset and Finch
Proc. of Int. Symp. on Fine Particle Processing
Vol. 2 ed. P. Somasundrian
Pub. Am. Inst. of Mining, Metallurgical and Petroleum Engineers Inc
New York 1980
p. 1217 – 1241
Using Blasius's solⁿ for flow.

$f_{max} = \frac{1}{4}\left(\left(\frac{N_L}{C}\right)^{4/5} - 1\right)$
$N_L = \frac{9}{\sqrt{2}}\left(\frac{a}{b}\right)\left(\frac{1}{Re}\right)^{0.5}\left(\frac{V_m}{V_0}\right)$ Nesset and Finch Loading Number
$Re = \frac{2a V_0 \rho_L}{\eta}$
$C = 2.5$

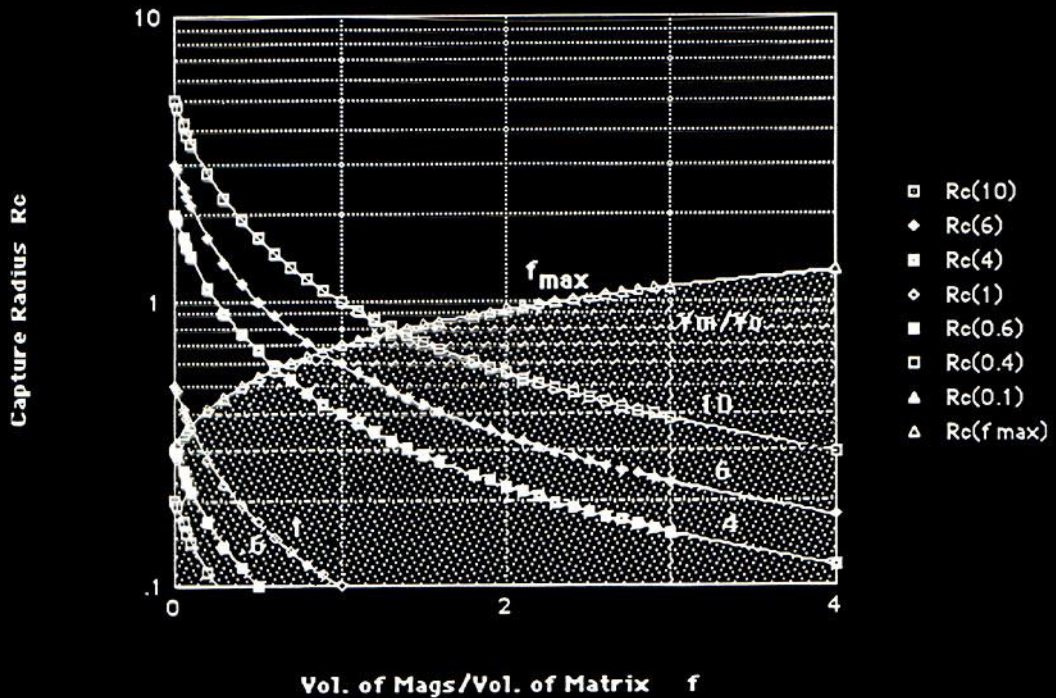
Ex/ $V_m/V_0 = 1$, $V_0 = .1\, m/s$ $a/b = 5$ $a = 50\mu m$ $\Rightarrow$
$Re = 10$, $N_L = 10.06$, $f_{max} = 0.51$, $R_c(f_{max}) = 0.16$ $R_c(f=0) = .5$



Indicates that particle retention is not possible if $V_m/V_0 < .62$

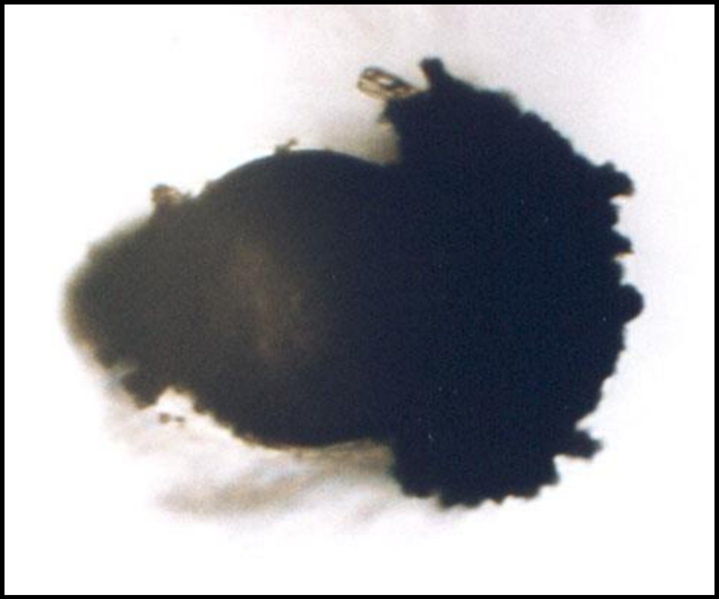
Ref Y.A. Liu and M.J. Oak
AIChE Journal 29 No 5
(Sept 1983) pp 771–9

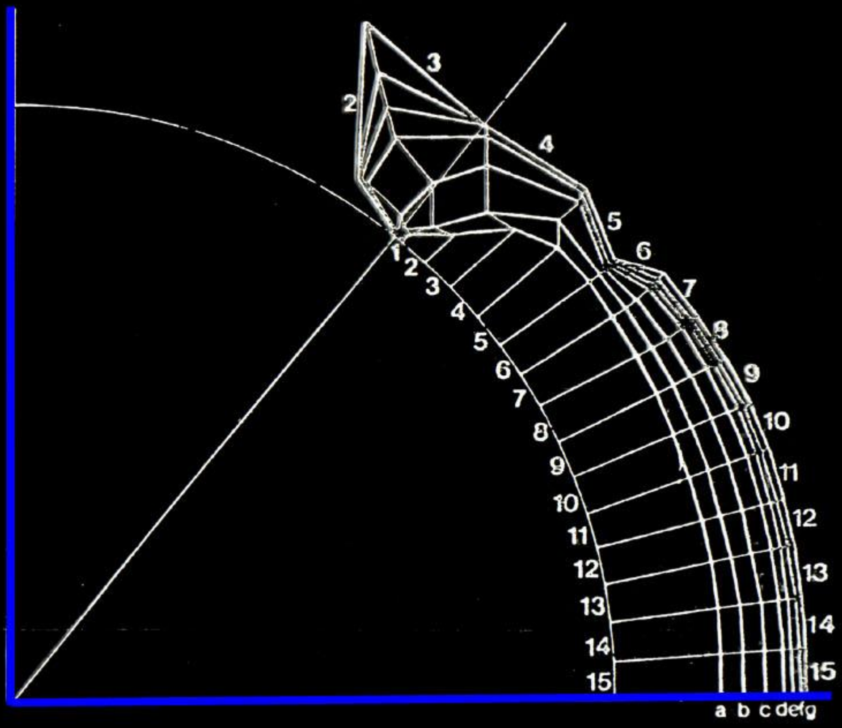
The capture radius R_c



The capture radius R_c versus f for various values of V_m/V_o and f_{max} according to the force-balance model [5] is shown as a solid line and labelled in the legend as R(Fmax).

The capture radius R_c





The capture radius R_c